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$$\begin{aligned}
\Delta &= \frac{2}{256\pi^4 r^8} \int_0^\pi \int_0^\theta \int_0^\pi \int_0^\phi \int_0^\pi \int_0^{2\pi} V \cdot 2\pi r^2 \sin\theta d\theta \cdot 2\pi r^2 \sin\beta d\beta \cdot 4r^2 \sin\theta \sin\varphi d\varphi d\lambda \\
&\quad \times 4r^2 \sin\theta \sin(\varphi - \psi) \sin\lambda \sin\psi d\psi d\rho \\
&= \frac{2r^8}{3\pi} \int_0^\pi \int_0^\theta \int_0^\pi \int_0^\phi \int_0^\pi \sin^5\theta (\cos\beta - \cos\theta) \sin\beta \sin^2\varphi \sin^2\psi \sin^2(\varphi - \psi) \sin\lambda d\theta d\beta \\
&\quad \times d\varphi d\psi d\lambda \\
&= \frac{4r^8}{3\pi} \int_0^\pi \int_0^\theta \int_0^\pi \int_0^\phi \sin^5\theta \sin\beta (\cos\beta - \cos\theta) \sin^2\varphi \sin^2\psi \sin^2(\varphi - \psi) d\theta d\beta d\varphi d\psi \\
&= \frac{r^8}{6\pi} \int_0^\pi \int_0^\theta \int_0^\pi \sin^5\theta \sin\beta (\cos\beta - \cos\theta) (3\varphi \sin^2\varphi - 2\varphi \sin^4\varphi - 3\sin^5\varphi \cos\varphi) d\theta d\beta d\varphi \\
&= \frac{r^8}{16\pi} \int_0^\pi \int_0^\theta \sin^5\theta \sin\beta (\cos\beta - \cos\theta) d\theta d\beta \\
&= \frac{\pi r^8}{32} \int_0^\pi \sin^5\theta (2 - 2\cos\theta - \sin^2\theta) d\theta = (4\pi r^8/105) = \frac{1}{8} \text{ of volume of sphere.}
\end{aligned}$$

133. Proposed by L. C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.

A circle of unknown radius is drawn with its center at the vertex of a given parabola, and has its greatest area when its circumference passes through the focus of the parabola. Required the average area common to the circle and parabola.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $y^2 = 4ax$, $x^2 + y^2 = r^2$ be the equation to the parabola and circle, respectively, $\Delta =$ average area, $x' = \sqrt{(r^2 + 4a^2)} - 2a$.

$$\begin{aligned}
\therefore \Delta &= 2 \int_0^a \left[2 \int_0^{x'} \sqrt{(ax)} dx + \int_{x'}^r \sqrt{(r^2 - x^2)} dx \right] dr / \int_0^a dr \\
&= \int_0^a \left[2 \int_0^{x'} \sqrt{(ax)} dx + \int_{x'}^r \sqrt{(r^2 - x^2)} dx \right] dr = \frac{1}{6a} \int_0^a \left[3\pi r^2 + 4\sqrt{(a)} [\sqrt{(r^2 + 4a^2)} \right. \\
&\quad \left. - 2a]^\frac{3}{2} - 6r^2 \sin^{-1} \left(\frac{\sqrt{(r^2 + 4a^2)} - 2a}{r} \right) \right] dr = \frac{1}{6} \pi a^2 - \frac{1}{3} a^2 \sin^{-1}(\sqrt{5} - 2) \\
&\quad + \frac{2}{3a} \int_0^a [\sqrt{(r^2 + 4a^2)} - 2a]^\frac{3}{2} dr + \frac{1}{3a} \int_0^a \frac{r^2 [\sqrt{(r^2 + 4a^2)} - 2a]^\frac{3}{2} dr}{\sqrt{(r^2 + 4a^2)}}.
\end{aligned}$$

$$\text{Let } \sqrt{r^2 + 4a^2} = 2y - r, \quad \frac{a}{2}(\sqrt{5} + 1) = y'.$$

$$\therefore \Delta = \frac{1}{8}\pi a^2 - \frac{1}{8}a^2 \sin^{-1}(\sqrt{5} - 2) + \frac{\sqrt{a}}{3a} \int_a^{y'} \frac{(3y^2 + 3a^2 + 2ay)(y - a)^3 dy}{y^3 \sqrt{y}}$$

$$= \frac{1}{8}\pi a^2 - \frac{1}{8}a^2 \sin^{-1}(\sqrt{5} - 2) + \frac{\sqrt{a}}{3a} \int_a^{y'} (3y\sqrt{y} - 7a\sqrt{y} + 6a^2/\sqrt{y} - 6a^3/y\sqrt{y}$$

$$+ 7a^4/y^2\sqrt{y} - 3a^5/y^3\sqrt{y}) dy = \frac{1}{8}\pi a^2 - \frac{1}{8}a^2 \sin^{-1}(\sqrt{5} - 2) + \frac{2}{45}a^2 [(86 - 13\sqrt{5})$$

$$\sqrt{\frac{\sqrt{5} + 1}{2}} - 128 + 11(11 - 2\sqrt{5}) \sqrt{\frac{\sqrt{5} - 1}{2}}].$$

Solved with a different result by the *PROPOSER*.

134. Proposed by G. B. M. ZERE, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

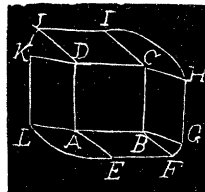
An ellipse, semi-axes a, b , is placed on a square, side c . Find the chance that center of ellipse is on the square.

Solution by the *PROPOSER*.

Let $ABCD$ be the square, and let the ellipse move parallel to itself so that the major axis makes an angle θ with the side BC . Then the center of the ellipse will describe the area $GHIJKLEFG$.

Let $(a^2 - b^2)/a^2 = e^2$, $p = \text{chance}$. The perpendicular distance from BC to $GH = a\sqrt{1 - e^2 \cos^2 \theta}$, the perpendicular distance from DC to $IJ = a\sqrt{1 - e^2 \sin^2 \theta}$.

$$\text{Area} = \pi ab + c^2 + 2ac [\sqrt{1 - e^2 \sin^2 \theta} + \sqrt{1 - e^2 \cos^2 \theta}] = u.$$



$$1/p = \int_0^{1/\pi} u d\theta / \int_0^{2\pi} c^2 d\theta = \frac{2}{\pi c^2} \int_0^{1/\pi} u d\theta. \quad \therefore 1/p = [\pi ab + c^2 + \frac{8ac}{\pi} E(e, \frac{1}{2}\pi)] / c^2.$$

$$\text{If } a=b, 1/p = (\pi a^2 + c^2 + 4ac) / c^2. \quad \text{If } a=b=c, 1/p = \pi + 1 + 4.$$

135. Proposed by LON C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.

If the line joining two points taken at random in the surface of a given circle be the diagonal of a square, the chance that the square lies wholly within the circle is $2 - 4/\pi$.

Solution by the *PROPOSER*.

Let MN be the line joining the random points M, N ; $MRNS$ the square; Q , its center; O , the center of the circle. Let the square move about the circle, so as to be within it, but in contact with it, the diagonal MN remaining parallel